Economic Modelling 28 (2011) 1279–1290

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Purchasing power parity and the long memory properties of real exchange rates: Does one size fit all?

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ARTICLE INFO

Article history:
Accepted 13 January 2011

JEL classification:
C12
C22
C32
F30
F31

Keywords:
Fractional Integration
Nonlinear modelling
Mean reverting process
Long-memory process

ABSTRACT

This paper examines the time series behavior of monthly bilateral real exchange rates (RER) on a comprehensive sample of 78 industrialized and developing countries, using the US Dollar, the UK Pound and the German Deutsche Mark as *numeraires*. We suggest a three step testing procedure based on recently introduced econometric techniques, in order to assess the mean-reverting properties of the RER and to address the question of whether real exchange rates follow a non linear process or a long memory process. The main results are as follows. Firstly, most of the bilateral real exchange rates under study are not mean-reverting. Secondly, the nonlinear ESTAR type adjustment is far from being prominent. Finally, only few bilateral RER exhibit true long memory mean-reverting properties.

1. Introduction

Most models of international trade and open economy rest on the hypothesis of purchasing power parity (PPP). At the aggregated level, this hypothesis implies that the nominal exchange rate should converge to the ratio of price levels between two countries, i.e. the real exchange rate (RER) should be a mean-reverting process.

The empirical validity of this PPP assumption remains one of the most active and controversial issues in international economics (Taylor, 2006; Taylor and Taylor, 2004). Empirical methodologies and results are mixed. Generally, the usual unit-root tests conclude that PPP does not hold during the post-Bretton Woods period (see Section 2 for a brief survey). Some potential reasons to explain this puzzle are first that countries under study have very heterogeneous exposure to foreign markets, different commercial links with the leading countries (such as the United States, the United Kingdom, or Germany) and have experienced a variety of exchange rate regimes during the last thirty years. Secondly, it may also be that usual testing techniques are inadequate in presence of non-standard dynamics, such as nonlinearity, structural instability, or long memory processes.

This article addresses these empirical difficulties to check PPP during the post-Bretton Woods period in two ways. First, we use a broader set of countries than the set considered in the literature: we thus consider monthly data on 78 CPI-based bilateral real exchange rates of industrialized and developing economies, over the period 1970–2006. For each currency, we consider three bilateral nominal exchange rates, the *numeraire* being alternatively US Dollar, UK Pound and German Mark. It can be seen that the RER behavior does not only depend on the period and the country under study but also on the *numeraire* used in computing the bilateral RER: the PPP hypothesis is more likely to occur for countries commercially linked or geographically close to the countries of which the currency is taken as *numeraire*. Second, we use recent econometric techniques to detect a long-memory process or a short-memory process with structural breaks. Our sequential testing strategy consists of three steps. First, we test for mean-reversion using the FELW estimator of long-memory parameter and drop from the sample each series which does not follow a mean-reverting process. Second, we determine whether the mean-reverting processes are stationary mean-reverting or not. Third, among non-stationary-mean-reverting processes, we discriminate between true long-memory processes and short-memory processes contaminated by abrupt changes in level. Finally, we compute impulse-response functions in order to evaluate half-lives for those true long memory mean-reverting bilateral RER.

The main results are as follows. Firstly, most of the bilateral RER appear to be non mean-reverting processes. Secondly, the nonlinear

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doi:10.1016/j.econmod.2011.01.005

Exponential Smooth Transition Auto-Regressive (ESTAR) type adjustment is far from being prominent and, finally, only few bilateral RER exhibit true long memory mean-reverting properties. For these true long-memory processes, the half-lives are found to lie between 1 month and 6 years.

The remainder of the paper is as follows. In Section 2, the empirical literature on PPP is briefly reviewed. Section 3 considers the econometric methodologies used in the paper. Section 4 reports the empirical results and Section 5 concludes.

2. Controversies as to the PPP hypothesis in the empirical literature: a selected review

The breakdown of the Bretton Woods system at the beginning of the 1970s stimulated a large flow of research about the long-run equilibrium level of the RER. On the theoretical side, most of macroeconomic models assume PPP in the short run (Frenkel, 1976) or, at least, in the long-run (Dornbusch, 1976). However, on the empirical side, the validity of the PPP hypothesis remains one of the most active and controversial issues in international economics (Taylor, 2003, 2006; Taylor and Taylor, 2004).

2.1. The two PPP puzzles

Since the volatility of nominal exchange rates appears to be more pronounced than the volatility of prices, empirical evidence overwhelmingly led to the PPP being rejected as a short run model of exchange rate. In order to test the validity of the PPP hypothesis as a long run relationship, early studies used standard testing methodologies such as the augmented Dickey–Fuller (ADF) test for a unit root in the real exchange rate. In logarithmic form, the RER q_t is defined as:

$$q_t = s_t - p_t + p_t^*, \tag{1}$$

where s_t is the log of the bilateral nominal exchange rate between the domestic and the foreign country, p_t and p_t^* are respectively the log of the domestic and foreign country price levels. As defined in Eq. (1), the RER measures the deviation from PPP: under long-run PPP, the logarithm of the RER must display reversion towards zero (after appropriate scaling).

According to the ADF-test, the RER is supposed to follow a linear autoregressive model that can be reparametrized as:

$$\Delta q_t = \rho q_{t-1} + \sum_{j=1}^{k-1} a_j \Delta q_{t-j} + a_0 + e_t, \tag{2}$$

where $\Delta q_t = q_t - q_{t-1}$ and e_t is a white noise disturbance. Under the null hypothesis H_0: $\rho = 0$, the RER contains a unit-root and displays no mean-reversion towards the PPP equilibrium. Using univariate unit root tests, numerous studies of RER in industrialized countries prove incapable of rejecting the null hypothesis for the post-Bretton Woods period when the US dollar is taken as *numeraire* (see e.g. the numerous references cited in Taylor (2006)).[1] Notably, some other studies find support for PPP when RER is expressed *vis-à-vis* the German Mark (Chowdhuri and Sdogati, 1993; Cheung and Lai, 1998, 2000) or for high inflation countries (e.g. Choudhry et al. (1991) which use multivariate cointegration analysis). The non-rejection of the unit root hypothesis in the RER is known as the first PPP puzzle (see e.g. Taylor et al. (2001)), since it questions one of the most popular intuitions among economists (Rogoff, 1996).

The second PPP puzzle highlighted in the literature reflects the fact that, among the studies which conclude in favor of RER mean-reversion, the empirical measurement of the half-lives of deviations from PPP is around three to five years (Rogoff, 1996). This high degree of persistence is at odds with the implications of sticky-price models of open economies, which imply that the half-life of a shock to the RER should be less than two years.[2]

Confronted by these two puzzles, three different processes are considered in the empirical literature to model the dynamic behavior of RER.

2.2. Linear autoregressive model

The first one is the linear autoregressive model (as described in Eq. (2)). Following Engle and Granger (1987) terminology, if the RER is found to follow a $I(0)$ – or stationary – process instead of a $I(1)$ – or nonstationary – process, it exhibits a *geometrical* reversion towards the long run equilibrium after a shock. In this case, PPP holds at least in the long run. However, the second puzzle is not solved when autoregressive roots are found to be close to one.

2.3. Nonlinearity and structural breaks

The second kind of process considered to model the RER is constituted by various forms of nonlinear models. Most of the literature consider alternatively the band-threshold autoregressive (TAR) model (e.g. Obstfeld and Taylor (1997)), the ESTAR process (e.g. Taylor et al. (2001)), or the Markov regime switching model (e.g. Kanas (2006)).

These nonlinear models belong to the more general class of structural break models, as illustrated by simple examples (see e.g. Park and Shintani (2005) and Cerrato et al. (2010) for some extensions). Following the ADF Eq. (2), nonlinear and structural break models can be represented by:

$$\Delta \tilde{q}_t = \beta \tilde{q}_{t-1} \rho(z_t, \theta) \sum_{j=1}^{k-1} a_j \Delta \tilde{q}_{t-j} + e_t, \tag{3}$$

where $\tilde{q}_t$ is the log of the RER expressed in deviations from the mean, $\rho(z_t,\theta)$ is the transition function, z_t is a transition variable and (β,θ) a set of parameters (assuming $\beta<0$ and $\theta>0$), both influencing the value of persistence. For instance, in the Self-Exciting Threshold Auto-Regressive (SETAR) model, the transition variable is the (demeaned) RER with lag delay $d \geq 1$, i.e. $z_t = \tilde{q}_{t-d}$, and the transition function can be given as $\rho(z_t,\theta) = 1\{\tilde{q}_{t-d} \leq -\theta\} + 1\{\tilde{q}_{t-d} \geq \theta\}$. Conversely, in the ESTAR model, the transition function is given by $\rho(z_t,\theta) = 1 - exp\left(-\theta \tilde{q}_{t-d}^2\right)$.

In these SETAR and ESTAR cases, the mean reversion occurs only when the size of the deviation from the long-run equilibrium exceeds a given threshold. For instance, considering the SETAR case, in the central regime $-\theta < \tilde{q}_{t-d} \leq \theta$, the RER follows an $I(1)$ process since $\rho(z_t,\theta) = 0$, while in the outer regimes ($\tilde{q}_{t-d} \leq -\theta$ or $\tilde{q}_{t-d} \geq \theta$) it follows an $I(0)$ mean-reverting process. In the ESTAR model, the transition between regimes is smooth, with a transition function bounded between 0 and 1. When the RER deviation from the mean is null ($\rho(z_t,\theta) = 0$), the RER follows an $I(1)$ process, and the transition towards the outer regime ($\rho(z_t,\theta) \neq 0$) occurs when the RER deviates from his mean ($\tilde{q}_{t-d} \neq 0$). In the latter case, the RER follows an $I(0)$ process and the speed of transition towards the outer regime increases with the value of θ.Whilst globally mean-reverting, these nonlinear processes follow a near random walk behavior for sufficiently small deviations from PPP. The ESTAR process becomes increasingly mean-reverting with the size of the deviation from the

[1] However, the literature has often pointed out the low power of standard unit-root tests over short time spans of data. Long-span or panel-data studies partly address this criticism.

[2] Some authors have recently discussed the appropriateness of usual measures of half-lives and have suggested some alternative measures for assessing the persistence in real exchange rates (see e.g. Chortareas and Kapetanios (2004), El-Gamal and Ryu (2006)).

equilibrium. These kinds of processes can be justified by theoretical models introducing transaction costs in international arbitrage[3] (Dumas, 1992; Sercu et al., 1995 among others): the costs of trading goods induce persistent deviations from PPP as long as these deviations are sufficiently small relative to the cost of trading.

In the Markov switching model, the transition function can be defined as $\rho(z_t,\theta)=s_t$ were the transition variable $z_t=s_t$ is an unobservable latent variable following a two-state first-order Markov process, taking the value 0 or 1. The probability of transition from regime i to regime j is constant and defines a 2×2 matrix whose elements are given by $p_{ij}=Pr(s_t=j|s_{t-1}=i)$ for $i=0,1$ and $j=0,1$. For instance, when $s_t=0$ the RER follows an $I(1)$ process, while when $s_t=1$ it follows an $I(0)$ process (given that $\beta<0$).

These various processes belong to the general class of structural break models since the regression coefficient ρ (Eq. (2)) is time-varying.[4]

A first implication of these specifications is related to the first PPP puzzle. Indeed, since the seminal work of Perron (1989), it has been widely recognized that the usual linear unit root tests are biased towards not rejecting a false null of a unit root when the true process is non-linear or when structural breaks are present. To address this problem, recent researches use nonlinear techniques instead of the standard unit root tests. For example, Kapetanios et al. (2003) have developed the KSS test, a new unit root test statistic more powerful against a stationary ESTAR process than the standard ADF test.[5] Since linear unit root tests might not be able to discriminate between unit root and nonlinear processes, the KSS test is designed to detect evidence of nonlinear mean reversion for most cases where linear unit root tests fail.[6]

The second implication is related to the second PPP puzzle. The RER half-life – in non-linear or structural break models – depends on both the size of the shock and the initial conditions. For instance, Taylor et al. (2001) found that the ESTAR model is consistent with half-lives lying in the range of three to five years for small and near PPP equilibrium shocks. Conversely, the speed of mean reversion is found to be substantially faster for larger shocks.

Since non linearity is likely to solve the two PPP puzzles, numerous recent papers (Cerrato et al., 2010; Dufrénot et al., 2006, 2008; Kiliç, 2009; McMillan, 2009 among many others) have emphasized the nonlinear mean reverting hypothesis. However, as it is discussed in the following section, nonlinear models may be spuriously selected, in so far as they can easily be confused with long memory processes.

2.4. Long-memory

Finally, the third kind of process used to model the RER dynamics is the long memory case. Granger and Joyeux (1980) showed the usefulness of distinguishing between integer and fractional integration. The order of integration of a so-called fractionally integrated process is a non-integer number, usually denoted by d: if $d>0$ the process has long memory properties; it is stationary if d lies in the interval (0,0.5) and mean reverting if $d<1$. Therefore a time series can be neither $I(0)$ nor $I(1)$, questioning the relevance of the usual unit-root tests to detect stationarity. In this respect, Lee and Schmidt (1996) have shown that the KPSS test – initially developed to test for an $I(0)$ null hypothesis against an $I(1)$ alternative – can be relevant to distinguish short memory from long memory stationary processes, since this test is shown to be consistent against an $I(d)$ alternative.

The long-memory hypothesis may be relevant in the PPP debate since temporal aggregation[7] (Taylor, 2001) or cross-sectional aggregation[8] (Imbs et al., 2005) are found to induce a positive bias in the computed aggregate half-lives. In the long-memory literature, it has been demonstrated (see for instance Haubrich and Lo (1989) and the discussion in Diebold and Inoue (2001)) that aggregation may be a source of long-term dependence. Fractional integrated processes exhibit a *hyperbolic* reversion towards the long-run equilibrium after a shock, provided that $0<d<1$: the mean-reversion rate is a decreasing function of time since the shock. In consequence, the usual half-life measure should be completed with more general m-lives computations.

Papers testing long-memory in the field of the PPP hypothesis (Baum et al., 1999; Cheung and Lai, 2005; Gil-Alana, 2000; Gil-Alana and Toro, 2002; Holmes, 2002; Villeneuve and Handa, 2006) report very mixed results.

Moreover, some papers (Diebold and Inoue, 2001; Granger and Hyung, 2004; Smith, 2005) have shown that the estimation of the long memory parameter d may be biased in presence of structural changes or regime switches. Conversely, Granger and Hyung (2004) underline the fact that fractional integration causes multiple breaks in the series (depending on the value of d) to be detected spuriously by usual estimation methods.

Since the presence of structural breaks may generate spurious long memory, the crucial question is to determine whether RER follows a true long memory process or a nonlinear mean reverting process such as ESTAR, time-varying STAR (Sollis, 2008) or Markov switching processes (Bergman and Hansson, 2005; Kanas, 2006). In this respect, Perron and Qu (2004, 2010) propose a simple testing procedure to distinguish between short memory contaminated by structural change and true long memory process. To our knowledge, this new test has not yet been applied on RER series except in McMillan (2009) where only five monthly real exchange rates against the US dollar were considered, namely, Canada, Germany, Japan, Switzerland and the UK.

These different cases have not been considered in a unified framework in the literature. The contribution of this article is to propose a general testing framework to discriminate between these competing models. To investigate more precisely the mean-reverting behavior of RER (i.e. the PPP hypothesis), this paper considers a variety of different tests, some of which have only recently been developed: the Kwiatowski et al. (1992) test (KPSS), in order to test for a $I(0)$ null hypothesis against a $I(d)$ alternative, the Kapetanios et al. (2003) test (KSS) to analyze nonstationarity against a stationary ESTAR process, the Robinson's (1994) test to detect fractional integration and the Perron and Qu (2004, 2010) to discriminate between a true long-memory process and a short memory process contaminated by structural changes in level. Finally, we compute impulse-response functions in order to evaluate half-lives for those true long memory mean-reverting bilateral RER.

3. Econometric methodology

3.1. Estimation of the long memory parameter

Concerning the estimation methods of the long memory parameters d, there exist different techniques (Geweke and Porter-Hudak, 1983; Robinson, 1995, among others). Here, we use the Feasible Exact Local Whittle (FELW) estimator developed by Shimotsu (2006)[9]. It is an

[3] Others potential sources of nonlinearity in real exchange rates are suggested in the recent literature (Kilian and Taylor, 2003; Sarno and Taylor, 2001).

[4] However, in the PPP literature, structural breaks are more often characterized by temporary changes in the mean of the RER (see e.g. Christopoulos and Leon-Ledesma (2010)).

[5] Other testing methodologies are provided by Bec et al. (2004), Park and Shintani (2005) and Kruse et al. (2009).

[6] For instance, using monthly real effective exchange rates for 52 countries over the period 1994–2007, Bahmani-Oskooee et al. (2008) conclude that 11 currencies are stationary (at the 10 percent level of significance) according to the KSS test whereas only 5 currencies were supposed to be stationary according to the ADF statistic (without trend).

[7] When the RER follows a first-order autoregressive at a higher frequency than that at which the data is sampled.

[8] Aggregating across different goods characterized with different speeds of reversion.

[9] We use the code available from K. Shimotsu at http://qed.econ.queensu.ca/faculty/shimotsu/programs/elwcode.zip.

extended version of the exact local Whittle (ELW) estimator proposed by Shimotsu and Phillips (2004, 2005, 2006), that is a semiparametric estimator generally giving a good estimation method for the memory parameter in terms of consistency and limit distribution, except in the case where the mean is unknown. To overcome this difficulty, Shimotsu (2006) extended the ELW estimator to the FELW estimator and showed that this estimator is consistent and has an $N(0,\frac{1}{4})$ limit distribution for $d\in(-\frac{1}{2},2)$.

3.2. Robinson's test of fractional integration

We know that a time series y_t follows an ARFIMA(p,d,q) (Autoregressive Fractionally Integrated Moving Average) process if

$$\Phi(L)(1-L)^d y_t = \mu + \Theta(L)\varepsilon_t, \tag{4}$$

where

$$\Phi(L) = 1-\phi_1 L-\ldots-\phi_p L^p, \Theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q, \tag{5}$$

L is the Backward shift operator i.e. $Ly_t = y_{t-1}$, and $\varepsilon_t \sim iid(0,\sigma^2)$. Different cases are possible, depending on the value of the long memory parameter d; for example, y_t is stationary and possesses shocks that disappear hyperbolically when $0<d<1/2$, whereas it is nonstationary and mean reverting for $1/2\leq d<1$.

We use the methodology elaborated by Robinson (1994) for testing unit root and other nonstationary hypotheses. Let us consider the null hypothesis defined by

$$H_0 : \theta = 0 \tag{6}$$

in the model given by:

$$y_t = \beta' z_t + x_t \tag{7}$$

and

$$(1-L)^{d+\theta} x_t = u_t \tag{8}$$

for $t = 1,2,\ldots$, where y_t is the observed time series, z_t is a $k\times 1$ vector of deterministic regressors, u_t is a (possibly weakly autocorrelated) $I(0)$ process, and d is a real parameter. Robinson (1994) proposes a Lagrange Multiplier (LM) statistic, called $\hat{r}$ (see Appendix 1 for details), and shows that it has a standard asymptotic distribution under some regularity conditions:

$$\hat{r} \underset{d}{\rightarrow} N(0,1) \text{ as } T\rightarrow\infty. \tag{9}$$

Thus, it is a one-sided test of $H_0:\theta=0$: we reject H_0 against $H_1:\theta>0$ if $\hat{r}>z_\alpha$ and against $H_1:\theta<0$ if $\hat{r}<-z_\alpha$, where the probability that a standard normal variate exceeds z_α is α.

This Robinson's (1994) test has been used in several papers in order to detect fractional integration (Caporale and Gil-Alana, 2007a, b,c; Gil-Alana and Nazarski, 2007), fractional integration with nonlinear models (Gil-Alana and Caporale, 2006; Cunado et al., 2007) and fractional integration with structural breaks (Caporale et al., 2007; Gil-Alana, 2008).

3.3. The KSS test

The KSS test elaborated by Kapetanios et al. (2003) aims to analyze nonstationarity under the null hypothesis against nonlinear but globally stationary ESTAR process under the alternative. The ESTAR model is given by

$$\Delta y_t = \phi y_{t-1}\left[1-exp\left(-\gamma y_{t-1}^2\right)\right] + \varepsilon_t \tag{10}$$

where γ is the smoothness parameter. A Taylor approximation of the transition function around $\gamma=0$ leads to the auxiliary regression:

$$\Delta y_t = \psi y_{t-1}^3 + u_t \tag{11}$$

where y_t is the demeaned or detrended data. The null and alternative hypotheses are: $\mathrm{H}_0:\psi=0$, $\mathrm{H}_1:\psi<0$ and the test statistic, whose critical values are given in Kapetanios et al. (2003), is written as

$$t_{\psi=0} = \frac{\hat{\psi}}{\sigma_{\hat{\psi}}}.$$

In the general case where the errors are autocorrelated, Eq. (11) is extended to:

$$\Delta y_t = \psi y_{t-1}^3 + \sum_{i=1}^{m} \rho_i \Delta y_{t-i} + u_t. \tag{12}$$

The lag length (m) is chosen by assessing the significance of the augmented coefficients.

In this paper, we estimate the $t_{\psi=0}$ statistic using only demeaned and not detrended, data: under the null, the real exchange rate follows a simple random walk.

3.4. Perron and Qu test

Perron and Qu (2004, 2010) develop a simple test based on the log periodogram estimator proposed by Geweke and Porter-Hudak (1983); they show how the distribution of this estimator is highly dependent on the number of frequencies used, especially when the data generating process is a stationary short memory process contaminated by structural changes in level. This test is thus helpful to distinguish structural change from long memory.

Let $\hat{d}_a$ (respectively $\hat{d}_b$) denote the log periodogram estimate of the memory parameter when $m_a=[T^a]$ (respectively $m_b=[T^b]$) frequencies are included in the regression. Under the null hypothesis of a stationary Gaussian fractionally integrated process, if $0<a<b<1$ and $a<4/5$, Perron and Qu (2004, 2010) demonstrate that the test statistic follows a Gaussian process under the null:

$$\sqrt{\frac{24[T^a]}{\pi^2}}\left(\hat{d}_a-\hat{d}_b\right) \xrightarrow{d} N(0,1).$$

To test whether the process is a true long memory and not a short-memory process with level shifts, they use this statistic with $a=1/2$ and $b=4/5$. They note that it is not sensitive to the value of d even if $d>1/2$, and is consistent against a short memory process with level shifts or a long-memory process with a strongly mean reverting component.

3.5. Impulse response functions and half-life analysis

One way to estimate the persistence of the different series is to fit an ARFIMA model to y_t and to estimate its impulse response function. By allowing the long memory parameter d to take non-integer values, the fractional model accommodates a broader range of low-frequency, mean-reverting dynamics than standard time series models.

The mean-reverting property holds if $d<1$ whereas the impact of a shock is known to persist forever in case of a unit-root process ($d=1$). This can be seen from the moving average representation for $(1-L)$ y_t:

$$(1-L)y_t = A(L)\varepsilon_t$$

with

$$A(L) = (1-L)^{1-d}\Psi(L)$$
$$= 1 + a_1L + a_2L^2 + \ldots.$$

and $\Psi(L) = \Phi(L)^{-1}\Theta(L) = 1 + \psi_1 L + \psi_2 L^2 + \ldots$, where Φ and Θ are defined in Eq. (5). The moving average coefficients a_j, and $j = 1, \ldots$, are referred to as the impulse responses and can be computed as follows:

$$a_j = \sum_{k=0}^{j} \frac{\Gamma(k+d-1)}{\Gamma(d-1)\Gamma(k+1)} \psi_{j-k},$$

where the (ψj) can be computed recursively:

$$\psi_0 = 1$$

$$\psi_j = \theta_j + \sum_{i=1}^{min(j,p)} \phi_i \psi_{j-i} \;\; if \;\; 1 \le j \le q$$

$$\psi_j = \sum_{i=1}^{min(j,p)} \phi_i \psi_{j-i} \;\; if \;\; j \ge q+1.$$

The cumulative impulse response function over j periods of time is given by

$$C_j = 1 + a_1 + \ldots + a_j \quad (13)$$

and it tracks the impact of a unit innovation at time t on the long run equilibrium relationship at time $t+j$. As $j \to \infty$, $C_\infty = A(1)$, measuring the long-run impact of the innovation (Campbell and Mankiw, 1987). Cheung and Lai (1993) show that for $d < 1, C_\infty = 0$, implying shock-dissipating behavior. Conversely for $d \ge 1, C_\infty \ne 0$, the effect of a shock will not die out. Mean reversion (i.e. $C_\infty = 0$) occurs as long as $d < 1$. A measure of persistence usually considered in the literature is the half-life, which indicates how long it takes after a unit shock to dissipate by half on the long-run equilibrium. The half-life can be computed from the C_j function as $t = h$ such that $C_h = 0.5$.

For ARMA models, an analytical expression for the half-life can be derived; for example, it is well known that the half-life of the $AR(1)$ model $y_t = \phi y_{t-1} + \varepsilon_t$ is given by $h = -log(2)/log(\phi)$. However, for the ARFIMA model, the half-life remains difficult to compute. This problem can be solved plotting the impulse response function and using linear interpolation.

4. Purchasing power parity: empirical analysis

4.1. The data

We consider the log-transformed monthly data over the period November 1970–August 2006, that is a sample size of 430 observations. The different series are the CPI-based bilateral real exchange rates of 78 countries against the US Dollar, the UK Pound and the German Deutsche Mark (Euro since January 1999). All the observations are computed using series obtained from the International Financial Statistics database. Appendix 2 contains the list of countries. Since this paper focuses mainly on the fractional integration hypothesis, detailed results are given only for countries for which the long memory parameter estimator is significantly lower than unity.

4.2. Empirical results

4.2.1. Integration analysis

For each time series we perform the following three steps (see Diagram 1):

Step 1. We compute the FELW estimator developed by Shimotsu (2006), $\hat{d}_w$. If $\hat{d}_w$ is significantly greater than 1 (in the sense that d_u, the upper bound of the 95% confidence interval, is greater than 1), then the series is not mean reverting and hence is discarded (Figs. 1–3 present the results for the whole dataset). According to this test, there remain 39 bilateral real exchange rates over 234[10]: for these series, the PPP holds since the real exchange rates are mean reverting and follow a linear or nonlinear, stationary or nonstationary process. In order to characterize more precisely the dynamics of the corresponding real exchange rates, we perform the next two steps.

Step 2. We perform the KPSS test for unit root, where the null hypothesis is the stationarity. If the null is rejected then the series may follow a long memory process with a parameter $0.5 \le d < 1$ (nonstationary and mean-reverting) or a stationary with structural change in the level (34 real exchange rates are found to be in this case).

Step 3. If the null of stationarity is rejected in step 2, we apply the KSS test to check if the series has a smooth change in the level (ESTAR) and the Perron and Qu (2004, 2010) test to decide if the series is a true long memory or a short memory contaminated by abrupt changes in the level. When the true long memory hypothesis is not rejected, the FELW estimator of Shimotsu (2006), $\hat{d}_w$, and the Robinson (1994) estimator d_R are used to estimate the corresponding degree of fractional integration.

The results of the KPSS test over the 39 series selected in Step 1 are depicted in Table 1: it is worth noting here that, in the first step, only 8 bilateral real exchange rates are found to satisfy the PPP hypothesis when the US Dollar is taken as *numeraire*, whereas this specific case is the most frequently used in the empirical literature. In the second step, the null hypothesis of the KPSS test is rejected for most of these series in level. The exceptions are Mexico with the US Dollar as *numeraire*, New Zealand, Belgium, France, and the Netherlands with the German Deutsche Mark/Euro as *numeraire*: in these five cases, the real exchange rates are thus found to be stationary mean-reverting processes, i.e. the PPP hypothesis is verified with a degree of integration $d < 0.5$.

In the third step, after discarding the five stationary mean-reverting series, we apply the KSS test on the 34 remaining series (Table 2): the null hypothesis of unit root appears to be firmly rejected (at the 1% level) in 10 cases over 34, against the ESTAR alternative: Bolivia, Brazil and Costa Rica with the US Dollar as *numeraire*; Australia, Bolivia and Mexico with the UK Pound as *numeraire*; Turkey, Bolivia, Brazil and Costa Rica with the German Deutsche Mark/Euro as *numeraire*. In these cases, we can infer that the real exchange rate is likely to follow:

(i) an ESTAR process (or, more generally, a short memory process contaminated with level shifts);
(ii) or a true long memory process (spuriously confused with an ESTAR process), since it is known that fractional integration causes many breaks in the series and can spuriously be confused with a short memory process with breaks.

In order to discriminate between these two hypotheses, we apply the Perron and Qu (2004, 2010) test: Table 3 shows the results of these tests, where p_{PQ} is the p-value.

Concerning the 34 real exchange rates selected in step 2, we thus find strong evidence in favor of fractional integration and mean reversion: the Perron and Qu test p-values conclude in acceptance that the process generating the series is a true long memory process without level shifts in nearly all cases, with the exception of the five

[10] The whole sample includes 234 real exchange rates series: 78 countries and 3 *numeraires*.

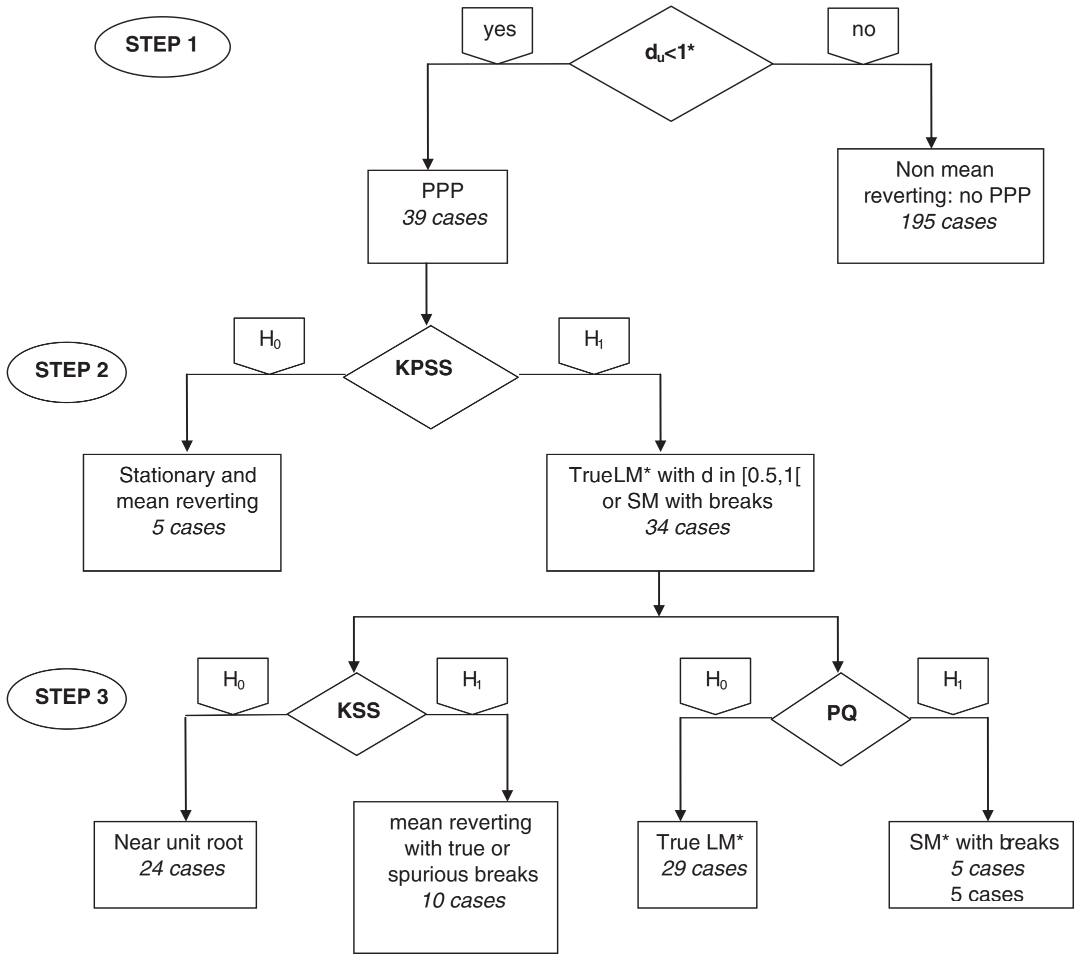

Diagram 1. The testing strategy. (*d_u: the upper bound of the 95% confidence interval corresponding to the FELW estimator developed by Shimotsu (2006), LM: long memory, SM: short memory).

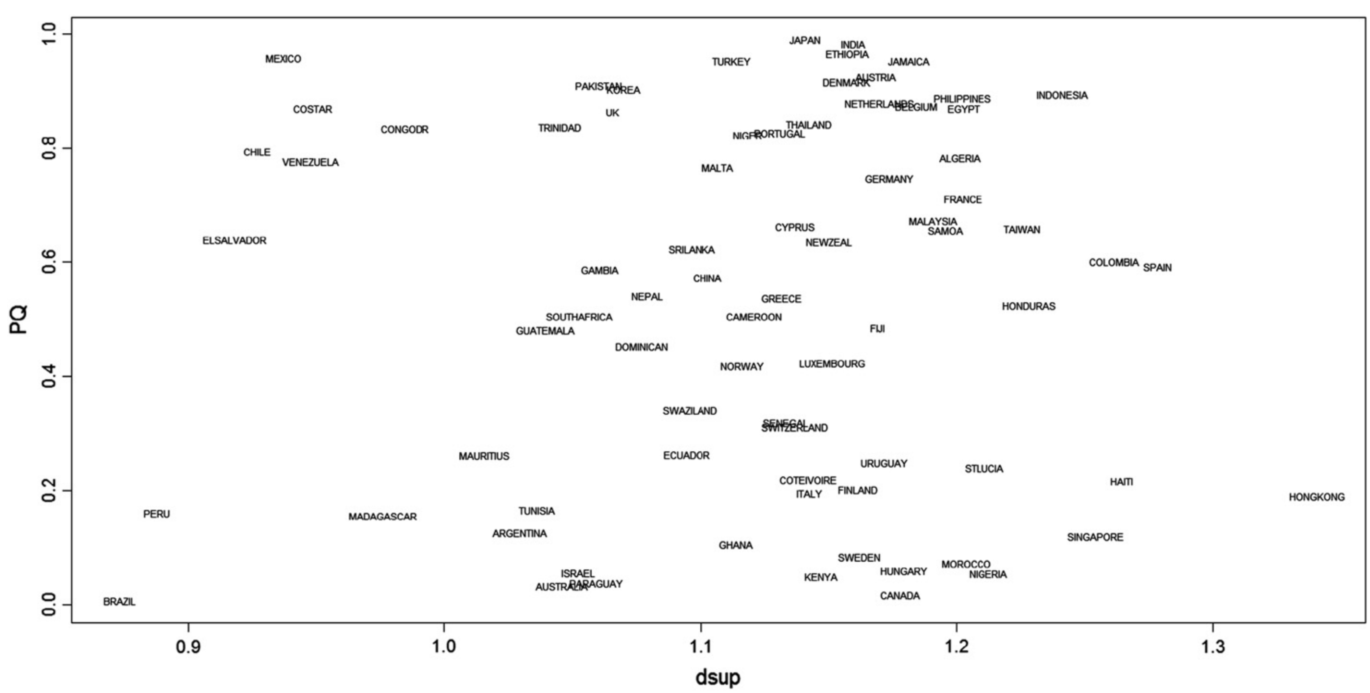

Fig. 1. Perron–Qu test significance levels and d-sup estimates of bilateral real exchange rates with respect to USA. PQ: *p*-value of the Perrun and Qu test. dsup: upper bound of the 95% interval on the long memory parameter (FELW estimator). Ommitted values: Bolivia: dsup = 0.664 PQ = 0.3769.

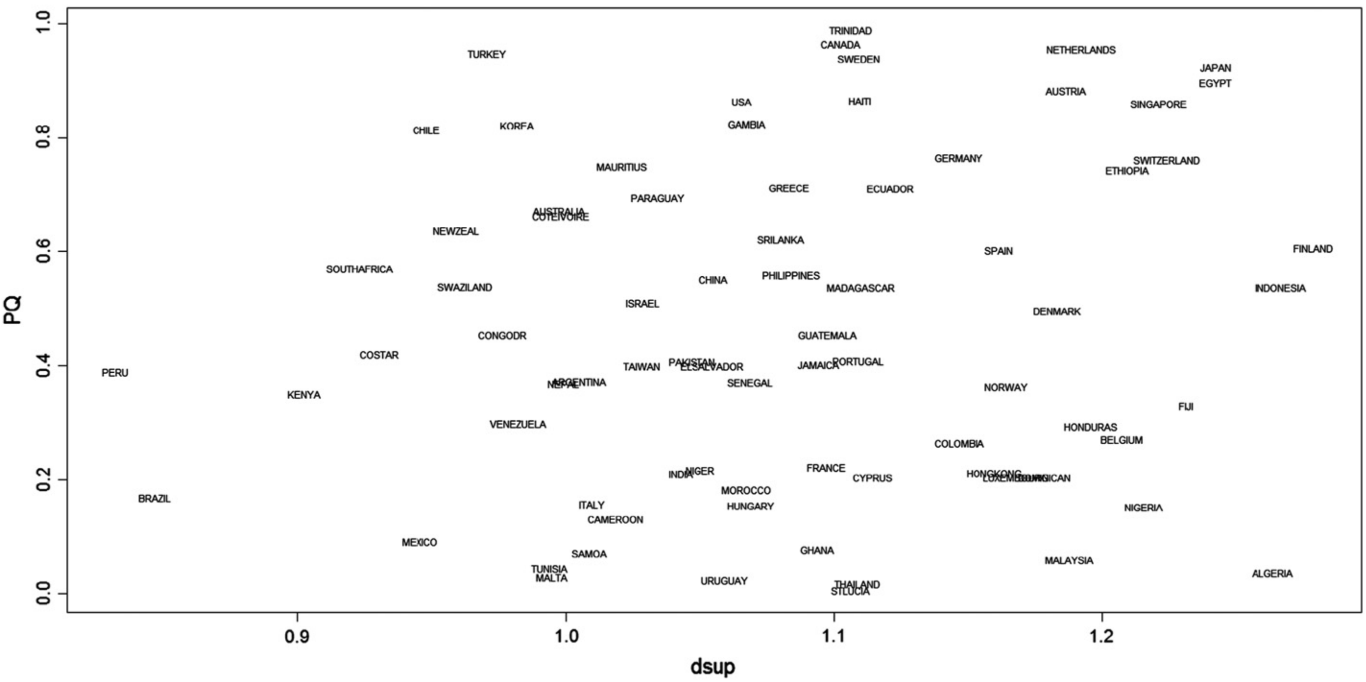

Fig. 2. Perron–Qu test significance levels and d-sup estimates of bilateral real exchange rates with respect to UK. PQ: *p*-value of the Perrun and Qu test. dsup: upper bound of the 95% interval on the long memory parameter (FELW estimator). Ommitted values: Bolivia: dsup = 0.7847 PQ = 0.4341.

following ones: Brazil for real bilateral exchange rate against the US Dollar, Malta, Mexico and Tunisia, for real bilateral exchange rate against the UK Pound, and Brazil, for real bilateral exchange rate against the German Deutsche Mark.

For these five countries, we have $p_{PQ}<0.10$: the value of p_{PQ} is thus consistent with a short memory process with level shifts and not a true long memory process. Taking into account the preceding KSS tests results, we can conclude that the ESTAR (or more generally the short memory with breaks) hypothesis is likely to be satisfied only in the following three cases: Brazil when the US Dollar is the *numeraire*, Mexico when the UK Pound is the *numeraire*, and Brazil when the German Deutsche Mark/Euro is the *numeraire*. This finding questions the relevance of the results obtained in most of the recent empirical literature (e.g. Taylor et al. (2001)).

The results of the Perron–Qu significance levels and the upper-bounds of the 95% intervals on the long memory parameters for the whole dataset are depicted in Figs. 1 to 3. These figures show clearly that most of the bilateral CPI-based real exchange rates follow a long memory – nonstationary and non-mean reverting – process, more particularly when the US Dollar is taken as *numeraire*.

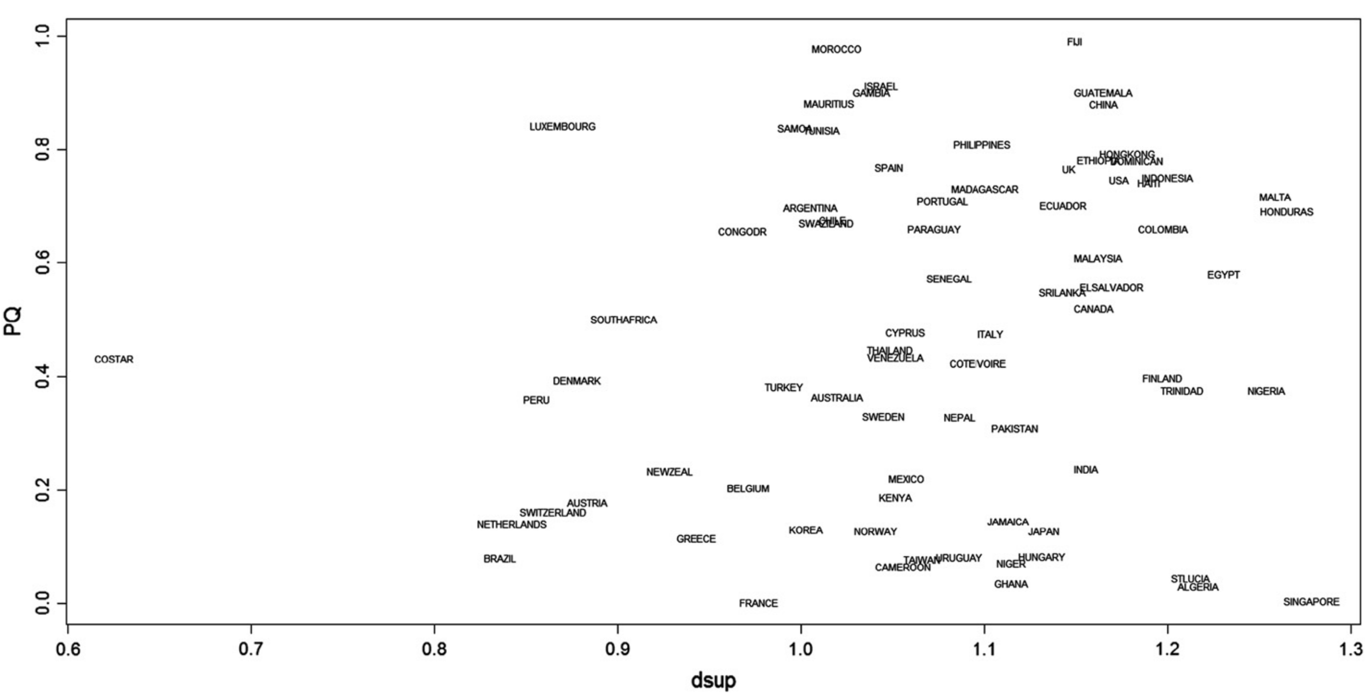

Fig. 3. Perron–Qu test significance levels and d-sup estimates of bilateral real exchange rates with respect to Germany. PQ: *p*-value of the Perrun and Qu test. dsup: upper bound of the 95% interval on the long memory parameter (FELW estimator). Ommitted values: Bolivia: dsup = 0.7816 PQ = 0.4093.

Table 1
Second step: the KPSS test (Real exchange rates in log).

With respect to US$		S_t		ΔS_t	
		$\hat{\eta}_\mu(6)$	$\hat{\eta}_\mu(12)$	$\hat{\eta}_\mu(6)$	$\hat{\eta}_\mu(12)$
South and Latin America	Bolivia	3.77	2.25	0.01	0.03
	Brazil	4.18	2.31	0.03	0.04
	Costa Rica	4.02	2.23	0.06	0.06
	El Salvador	5.70	3.15	0.02	0.03
	Mexico	0.59	0.34	0.03	0.03
	Peru	4.12	2.26	0.05	0.07
	Venezuela	2.18	1.24	0.04	0.05
Africa	Congo DR.	3.77	2.12	0.05	0.08
With respect to UK£		S_t		ΔS_t	
		$\hat{\eta}_\mu(6)$	$\hat{\eta}_\mu(12)$	$\hat{\eta}_\mu(6)$	$\hat{\eta}_\mu(12)$
Oceania	Australia	4.65	2.60	0.03	0.03
	New Zealand	1.95	1.11	0.03	0.04
Europe	Malta	5.48	3.01	0.08	0.10
	Turkey	3.58	1.99	0.06	0.08
South and Latin America	Bolivia	4.79	2.35	0.01	0.02
	Brazil	4.91	2.73	0.01	0.02
	Costa Rica	4.39	2.44	0.05	0.05
	Mexico	1.33	0.78	0.03	0.03
	Peru	3.59	1.99	0.05	0.08
	Venezuela	2.78	1.56	0.04	0.05
Africa	Tunisia	5.20	2.86	0.10	0.12
	Congo DR.	4.16	2.33	0.04	0.06
	Ivory Coast	3.93	2.18	0.05	0.06
	Swaziland	4.84	2.69	0.04	0.04
Asia	Nepal	5.73	3.13	0.08	0.09
With respect to German DM		S_t		ΔS_t	
		$\hat{\eta}_\mu(6)$	$\hat{\eta}_\mu(12)$	$\hat{\eta}_\mu(6)$	$\hat{\eta}_\mu(12)$
Oceania	New Zealand	0.32	0.19	0.02	0.02
Europe	Austria	5.15	2.82	0.10	0.15
	Belgium	0.88	0.52	0.02	0.03
	Denmark	4.46	2.50	0.02	0.02
	France	0.48	0.30	0.02	0.02
	Greece	3.26	1.87	0.16	0.21
	Luxembourg	2.38	1.38	0.05	0.08
	Netherlands	1.21	0.68	0.04	0.07
	Switzerland	4.50	2.51	0.15	0.19
	Turkey	2.27	1.28	0.09	0.11
South and Latin America	Bolivia	2.38	1.42	0.01	0.02
	Brazil	4.55	2.57	0.01	0.02
	Costa Rica	3.60	2.03	0.10	0.10
	Peru	4.62	2.55	0.06	0.09
Africa	Congo DR.	3.61	2.04	0.04	0.06
	South Africa	4.17	2.35	0.03	0.03

Note: S_t is the real exchange rate in log. $\hat{\eta}_\mu(l)$ is the $\hat{\eta}_\mu$ statistic of Kwiatowski et al. (1992) where l is the truncation parameter used in estimating the long run variance (see Kwiatowski et al. (1992) page 165); the critical values are: 0.347 (10%), 0.463 (5%) and 0.739 (1%).

Table 2
Third step: the KSS test (Real exchange rates in log).

With respect to US$		S_t	With respect to UK£		S_t
South and Latin America	Bolivia	−13.08	Oceania	Australia	−3.82
	Brazil	−4.42		New Zealand	−3.25
	Costa Rica	−5.66	Europe	Malta	−1.32
	El Salvador	−1.33		Turkey	−2.57
	Peru	−1.88	South and Latin America	Bolivia	−12.45
	Venezuela	−3.06		Brazil	−2.27
Africa	Congo DR.	−1.98		Costa Rica	−2.79
				Mexico	−3.91
				Peru	−2.10
				Venezuela	−3.04
			Africa	Tunisia	−0.77
				Congo DR.	−1.88
				Ivory Coast	−1.72
				Swaziland	−2.22
			Asia	Nepal	−1.83

With respect to German DM		S_t
Europe	Austria	−1.34
	Denmark	−1.33
	Greece	−1.62
	Luxembourg	−3.22
	Switzerland	−1.57
	Turkey	−3.64
South and Latin America	Bolivia	−11.62
	Brazil	−4.64
	Costa Rica	−5.06
	Peru	−1.79
Africa	Congo DR.	−3.17
	South Africa	−2.79

Note: S_t is the real exchange rate in log. KSS is the augmented Kapetanios et al. (2003) test performed using demeaned data; the critical values are: −2.66 (10%), −2.93 (5%), −3.48 (1%).

Table 3
Third step: the Perron and Qu (2004 and 2010) test.

With respect to US$		p_{PQ}	With respect to UK£		p_{PQ}
South and Latin America	Bolivia	0.37	Oceania	Australia	0.67
	Brazil	0.00		New Zealand	0.63
	Costa Rica	0.87	Europe	Malta	0.02
	El Salvador	0.63		Turkey	0.94
	Peru	0.16	South and Latin America	Bolivia	0.40
	Venezuela	0.77		Brazil	0.16
Africa	Congo DR.	0.83		Costa Rica	0.42
				Mexico	0.09
				Peru	0.38
				Venezuela	0.29
			Africa	Tunisia	0.04
				Congo DR.	0.45
				Ivory Coast	0.66
				Swaziland	0.53
			Asia	Nepal	0.36

With respect to German DM		p_{PQ}
Europe	Austria	0.17
	Denmark	0.39
	Greece	0.11
	Luxembourg	0.84
	Switzerland	0.16
	Turkey	0.38
South and Latin America	Bolivia	0.43
	Brazil	0.08
	Costa Rica	0.43
	Peru	0.36
Africa	Congo DR.	0.65
	South Africa	0.50

Note: p_{PQ} is the p-value of the Perron and Qu (2004 and 2010) test.

Finally, Table 4 collects the results of the fractional integration against respectively the US Dollar, the U.K. Pound and the German Deutsche Mark: $\hat{d}_W$ is FELW estimator of the fractional integration parameter; d_l and d_u are the lower and upper bounds of the 95% confidence intervals (m is chosen to be $m = T^{0.65}$ with T is the sample size and the lower and upper bounds on the long memory parameter are respectively −0.2 and 1.4); d_R is the value of the long memory parameter d corresponding to the absolute value of the minimum of the Robinson (1994) test statistic. The Robinson's (1994) test confirms the results of our first step selection procedure since $d_R < 1$ in all cases.

4.2.2. Half-life analysis

In order to model the exchange rates with ARFIMA models, i.e. to determine which might be the best way to characterize the behavior of the series, we start from a most general specification and we determine plausible specifications, depending on the significance of the parameters

Table 4
Results for fractional integration.

With respect to US$		d_l	$\hat{d}_w$	d_u	d_R
South and Latin America	Bolivia	0.38	0.52	0.66	0.62
	Brazil	0.59	0.73	0.87	0.62
	Costa Rica	0.67	0.81	0.94	0.87
	El Salvador	0.64	0.78	0.91	0.95
	Mexico	0.66	0.80	0.93	0.92
	Peru	0.61	0.75	0.88	0.82
	Venezuela	0.67	0.81	0.94	0.82
Africa	Congo DR.	0.70	0.84	0.98	0.77
With respect to UK£		d_l	$\hat{d}_w$	d_u	d_R
Oceania	Australia	0.72	0.86	0.99	0.82
	New Zealand	0.68	0.82	0.95	0.75
Europe	Malta	0.72	0.85	0.99	0.97
	Turkey	0.69	0.83	0.97	0.87
South and Latin America	Bolivia	0.50	0.64	0.78	0.67
	Brazil	0.57	0.70	0.84	0.62
	Costa Rica	0.65	0.79	0.93	0.87
	Mexico	0.67	0.80	0.94	0.95
	Peru	0.55	0.69	0.83	0.85
	Venezuela	0.70	0.84	0.98	0.85
Africa	Tunisia	0.71	0.85	0.99	0.77
	Congo DR.	0.69	0.83	0.97	0.77
	Ivory Coast	0.72	0.86	0.99	0.92
	Swaziland	0.68	0.82	0.96	0.95
Asia	Nepal	0.72	0.86	0.99	0.92
With respect to German DM		d_l	$\hat{d}_w$	d_u	d_R
Oceania	New Zealand	0.65	0.79	0.92	0.77
Europe	Austria	0.60	0.74	0.88	0.50
	Belgium	0.69	0.83	0.97	0.60
	Denmark	0.60	0.74	0.87	0.52
	France	0.70	0.84	0.97	0.65
	Greece	0.66	0.80	0.94	0.82
	Luxembourg	0.59	0.73	0.87	0.57
	Netherlands	0.56	0.70	0.84	0.47
	Switzerland	0.59	0.72	0.86	0.67
	Turkey	0.71	0.85	0.99	0.87
South and Latin America	Bolivia	0.51	0.64	0.78	0.72
	Brazil	0.56	0.69	0.83	0.62
	Costa Rica	0.62	0.76	0.89	0.87
	Peru	0.58	0.71	0.85	0.85
Africa	Congo DR.	0.69	0.82	0.96	0.77
	South Africa	0.62	0.76	0.90	0.95

Note: $\hat{d}_w$ is the FELW estimator developed by Shimotsu (2006) of the long memory parameter; d_l and d_u are the lower and upper bounds of the 95% confidence intervals (m is chosen to be $m = T^{0.65}$ with T is the sample size and the lower and upper bounds on the long memory parameter are respectively −0.2 and 1.4). d_R is the value of the long memory parameter d corresponding to the absolute value of the minimum of the Robinson (1994) test statistic.

and the usual diagnostic tests.[11] For true long-memory processes (p_{PQ}>0.10), we compute and plot impulse-response functions to evaluate half-lives (Figs. 4 to 6).

The Table 5 collects half-lives for bilateral real exchange rates with respect to the US Dollar, UK Pound and German Deutschemark. For some countries (Bolivia, Brazil, Costa Rica and Peru), the real exchange rates follow a long memory mean reverting process whatever the *numeraire* is. We pick the US Dollar as *numeraire* for these countries because they experiment a stronger commercial link with the USA than with European countries; therefore, the Table 5 concerns only 19 countries.

In the literature, when PPP holds, half-lives are generally found to be between 1.5 and 3 years (Cheung and Lai, 2000), in accordance with the price stickiness hypothesis. Gil-Alana and Toro (2002), by means of ARFIMA models, examined the real exchange rates in five developed countries and conclude that, for all countries, there is evidence of mean

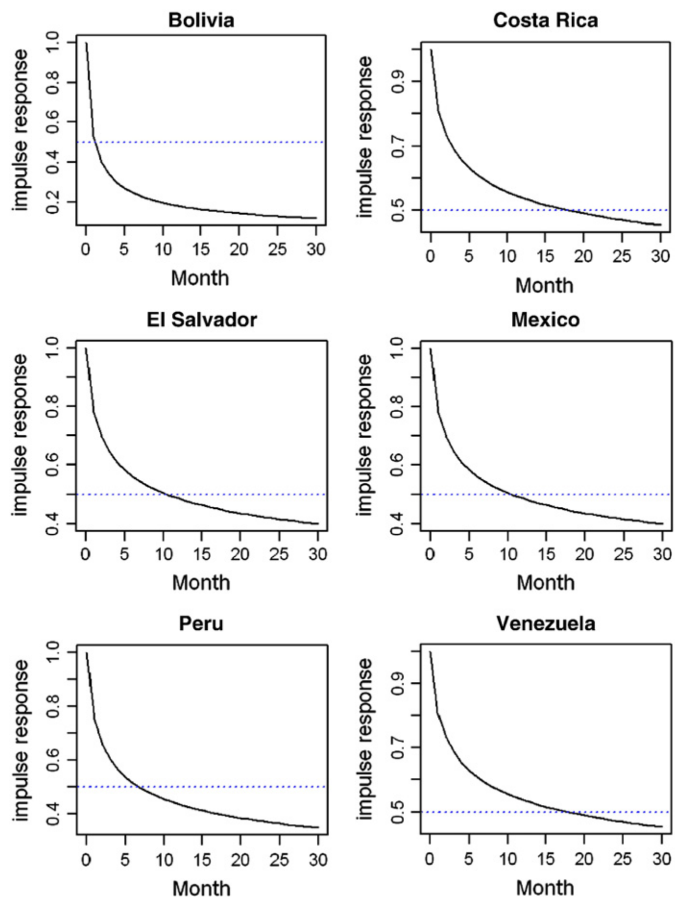

Fig. 4. The impulse response function of bilateral exchange rates with respect to USA.

reversion behavior; moreover, Italy and Japan seem to follow non-stationary processes while the exchange rates in the UK, Canada and France appear as stationary but with a long memory behavior. We show that the half-lives lie between 1 month and 6 years (Gil-Alana and Toro (2002) compute the half-lives in the case of five developed countries and conclude that the half-lives are above 7 years). Our results are in line with Cheung and Lai (2000) for Costa Rica, Venezuela, New Zealand, Turkey, Swaziland, Belgium. Faster convergence is observed for Bolivia, Peru, Austria, Denmark, Luxembourg, The Netherlands and Switzerland. Slower convergence is observed for Australia, The Ivory Coast and Nepal. According to Engel and Morley (2001), convergence to PPP level is mainly driven by nominal exchange rate changes. Even if the exchange rate regime does not influence the validity of PPP (Drine and Rault, 2008), it may affect the speed of convergence towards the long-run equilibrium. We can notice among faster convergence countries the presence of European Monetary Union countries (Luxembourg, Netherlands) and European countries that did not adopt the Euro as their currency (Denmark, which participates in the European Exchange Rate Mechanism II), or countries which do not belong to European Monetary Union (Switzerland). This may suggest that convergence to PPP is not so different between countries with flexible nominal exchange rate (Switzerland), semi-flexible nominal exchange rate (Denmark) and countries with fixed exchange rates (Netherlands, Luxembourg).

5. Concluding remarks

The empirical literature relating to the PPP hypothesis emphasizes two puzzles, namely the controversial findings about the mean reversion of RER towards the equilibrium and the high degree of persistence of the RER. In this respect, three main models (linear

[11] To save space, we display only the estimation of the fractional integration parameters. The results of the estimation of the ARMA components are available upon request.

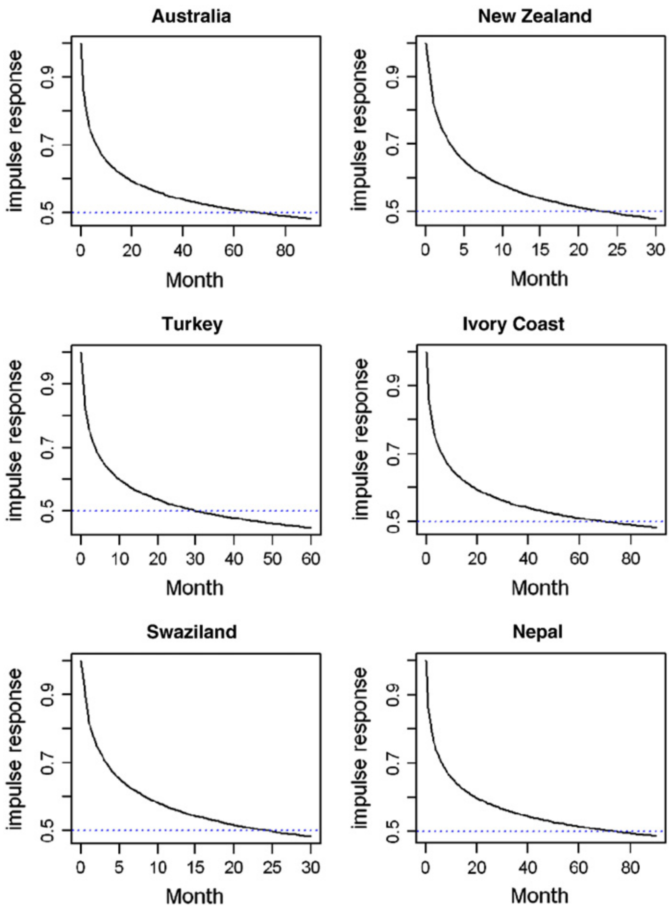

Fig. 5. The impulse response function of bilateral exchange rates with respect to UK.

autoregressive, nonlinear or structural break, and long memory) are prominent and lead to different conclusions: empirical evidence is mixed and appears to be generally specific to the chosen period and country. It is worthwhile noting that most studies focus on specific hypotheses and ignore the other competing models: the contribution of this article is to propose a general testing framework to discriminate between these different possibilities. In order to do so, we develop a unified sequential strategy proceeding in three steps.

Table 5
Half-life estimates of bilateral exchange rates (in month).

With respect to US$		With respect to UK£	
Bolivia	1.21	Australia	69.68
Costa Rica	18.11	New Zealand	24.06
El Salvador	10.48	Turkey	30.25
Mexico	14.56	Ivory Coast	69.68
Peru	6.73	Swaziland	24.18
Venezuela	17.73	Nepal	75.03
With respect to German DM			
Austria			6.40
Belgium			31.06
Denmark			5.94
Greece			15.03
Luxembourg			5.43
Netherlands			4.06
Switzerland			5.11

Note: The half-life is estimated by using a linear interpolation as follows: if k is such that $IRF[k] \geq 0.5 \geq IRF[k+1]$ then the linear approximation for the half-life estimate is given by $h = (0.5 - (k+1)IRF[k] + kIRF[k+1])/(IRF[k+1] - IRF[k])$.

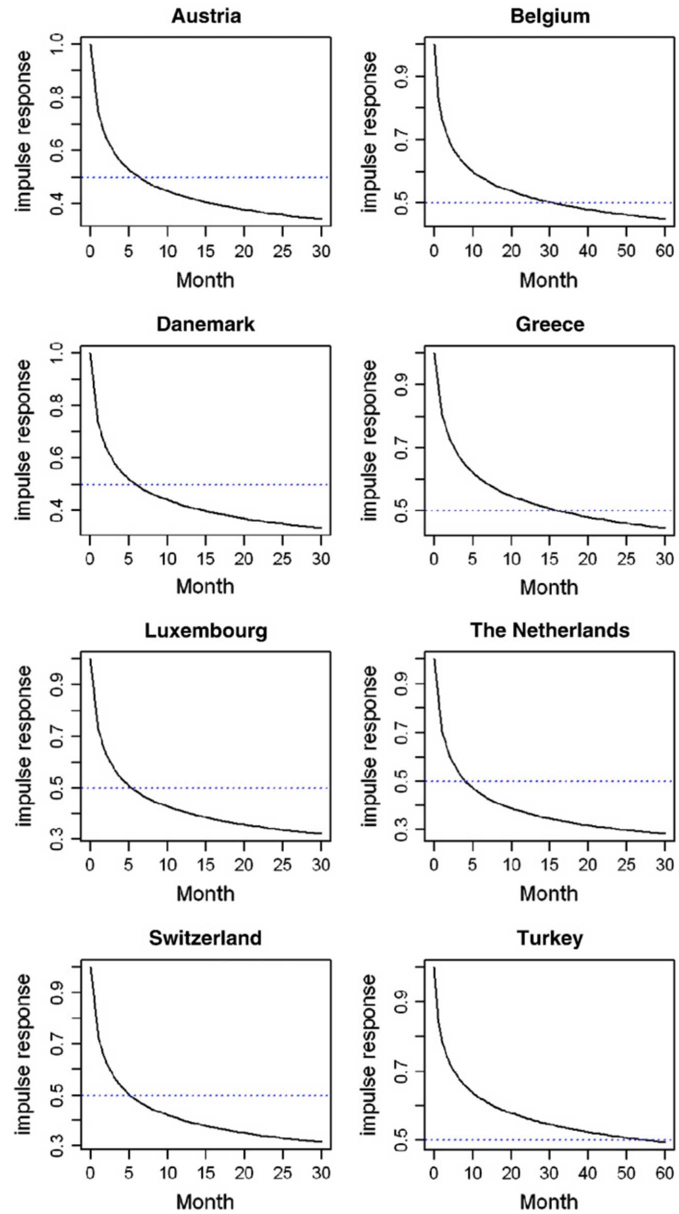

Fig. 6. The impulse response function of bilateral exchange rates with respect to Germany.

Based on 78 monthly CPI-based bilateral RER for industrialized and developing countries over the period 1970–2006, our results show that there is “no one-size fits all model”.

Firstly, the time series properties of RER appears to be not only period and country-specific but also specific to the *numeraire* used in the calculation of the bilateral RER: the PPP hypothesis would be more likely to occur for countries commercially linked or geographically close to the countries of which the currency is taken as numeraire. This question is left for future research.

Secondly, among the numerous countries under study, only few bilateral RER exhibit true long memory mean-reverting properties according to Robinson's and Perron and Qu's tests. In these cases, the half-lives are found to lie between 1 month and 6 years.

Finally, the ESTAR hypothesis, although extensively studied in the recent literature, is only confirmed in three cases: Brazil when the US Dollar is the numeraire, Mexico when the UK Pound is the numeraire, and Brazil when the German Deutsche Mark/Euro is the numeraire. We conclude that the nonlinear models are often spuriously selected, because the long memory processes may appear to have breaks, which can be confused with nonlinear processes.

Acknowledgments

The authors are deeply indebted to Luis A. Gil-Alana for providing various FORTRAN programs for the Robinson (1994), that they translated into GAUSS.

Appendix 1. The Robinson's (1994) LM statistic

The Lagrange Multiplier (LM) statistic proposed by Robinson (1994), $\hat{r}$, is given by:

$$\hat{r}=\left(\frac{T}{\hat{A}}\right)^{1/2}\frac{\hat{a}}{\hat{\sigma}^2} \tag{14}$$

where T is the sample size and

$$\hat{a}=\frac{-2\pi}{T}\sum_{j=1}^{T-1}\psi\left(\lambda_j\right)g\left(\lambda_j,\hat{\tau}\right)^{-1}I\left(\lambda_j\right),$$

$$\hat{\sigma}^2=\frac{2\pi}{T}\sum_{j=1}^{T-1}g\left(\lambda_j,\hat{\tau}\right)^{-1}I\left(\lambda_j\right),$$

$$\hat{A}=\frac{2}{T}\left[\sum_{j=1}^{T-1}\psi\left(\lambda_j\right)^2-\sum_{j=1}^{T-1}\psi\left(\lambda_j\right)\hat{\varepsilon}\left(\lambda_j\right)'\times\left(\sum_{j=1}^{T-1}\hat{\varepsilon}\left(\lambda_j\right)\hat{\varepsilon}\left(\lambda_j\right)'\right)^{-1}\sum_{j=1}^{T-1}\hat{\varepsilon}\left(\lambda_j\right)\psi\left(\lambda_j\right)\right],$$

$$\hat{\tau}=\arg\min_{\tau\in T^*}\hat{\sigma}^2,\quad \psi\left(\lambda_j\right)=\log\left|2\sin\frac{\lambda_j}{2}\right|,\quad \lambda_j=\frac{2\pi j}{T}$$

$$\hat{\varepsilon}\left(\lambda_j\right)=\frac{\partial}{\partial\tau}\log g\left(\lambda_j,\hat{\tau}\right),\quad g(\lambda,\tau)=\frac{2\pi}{\sigma^2}f\left(\lambda,\tau,\sigma^2\right);$$

f is the spectral density of u_t, T is a suitable set of R^k and $I(\lambda_j)$ is the periodogram of

$$\hat{u}_t=(1-L)^d y_t-\hat{\beta}'w_t \tag{15}$$

evaluated at λ_j with

$$w_t=(1-L)^d z_t$$

and

$$\hat{\beta}=\left(\sum_{t=1}^{T}w_t w_t'\right)^{-1}\sum_{t=1}^{T}w_t(1-L)^d y_t.$$

Note that σ^2 is generally no longer the variance of u_t, but rather the variance of the innovation sequence in a normalized Wold representation of u_t.

Appendix 2. List of countries

Algeria	El Salvador	Kenya	Samoa
Argentina	Ethiopia	Korea	Senegal
Australia	Fiji	Luxembourg	Singapore
Austria	Finland	Madagascar	South Africa
Belgium	France	Malaysia	Spain
Bolivia	Gambia	Malta	Sri Lanka
Brazil	Germany	Mauritius	St. Lucia
Cameroon	Ghana	Mexico	Swaziland
Canada	Greece	Morocco	Sweden
Chile	Guatemala	Nepal	Switzerland
China, PR	Haiti	Netherlands	Taiwan
Colombia	Honduras	New Zealand	Thailand
Congo	Hong Kong	Niger	Trinidad Tobago
Costa Rica	Hungary	Nigeria	Tunisia
Ivory Coast	India	Norway	Turkey
Cyprus	Indonesia	Pakistan	U.K.
Denmark	Israel	Paraguay	Uruguay
Dominican R.	Italy	Peru	Venezuela
Ecuador	Jamaica	Philippines	
Egypt	Japan	Portugal	

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